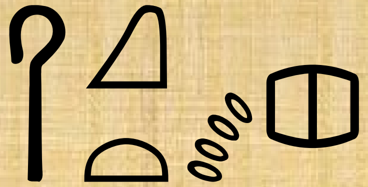
Ancient Egyptian Mathematics
Lesson 2: Fractions

Weighing of the *ib*
Papyrus of Ani

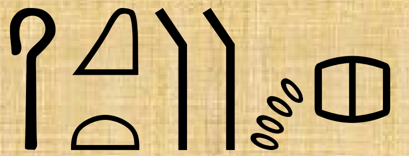
Egyptian Measures of Volume

Heqat (*ḥḳꜣ.t*): dry measure of volume

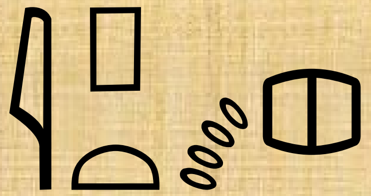


- Standard unit of measure for grain
- Equivalent to a modern 4.36 dry quarts (0.85 quart)s, 4.8 liters

Common Variants of Heqat



- Double Heqat (*ḥḳꜣ.tỉ*) = 2 Heqat
- Oipe (*ỉpt*) = 4 Heqat



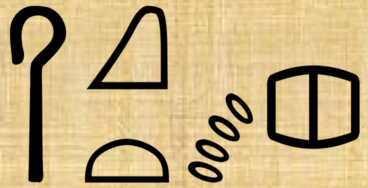
- Sack (*ẖꜥr*) = 10 Heqat

Liquid Measures of Volume

Jar (*ds*)

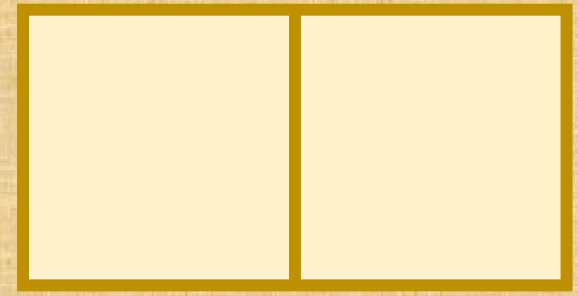
Hin (*hnw*)

Intuition for Egyptian Fractions

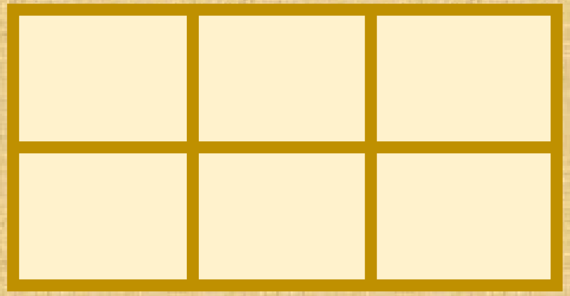
Heqat (*ḥḳꜣ.t*): dry measure of volume 

Pesu (*psw*): number of loaves of bread or jugs of beer obtained from one heqat of grain

1 Heqat of
1-Pesu Bread



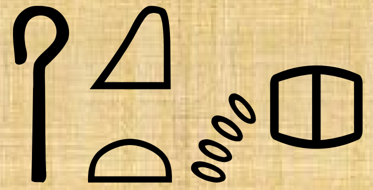
1 Heqat of
2-Pesu Bread



1 Heqat of
6-Pesu Bread

Intuition for Egyptian Fractions

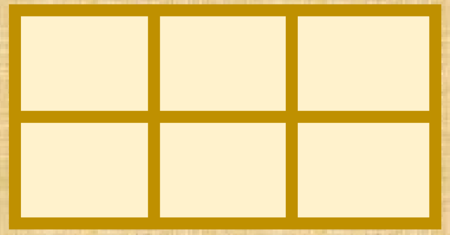
Heqat (*ḥḳꜣ.t*)**: dry measure of volume**



Pesu (*psw*)**: number of loaves of bread or jugs of beer obtained from one heqat of grain**

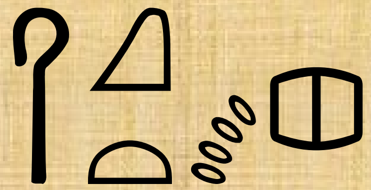
We formally do not need the concept of fractions!

By working with pesu, we count the number of loaves composing one heqat of grain. The emphasis is not on the relative size of each pesu compared to one heqat (where we would like to think of 1/6) but on there being *six* loaves within one heqat (6)



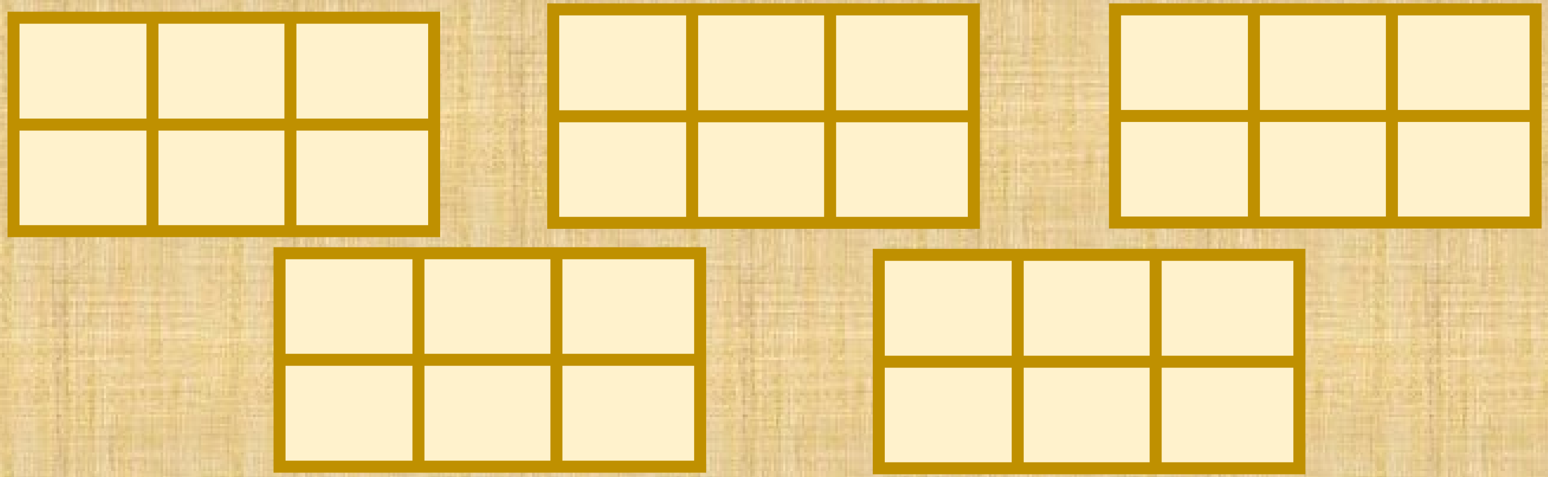
1 Heqat of
6 Pesu-Bread

Intuition for Egyptian Fractions

Heqat (*ḥḳ3.t*): dry measure of volume 

Pesu (*psw*): number of loaves of bread or jugs of beer obtained from one heqat of grain

Ostraca Question: How many loaves of 6-pesu bread can be made out of 5 heqat?



30 loaves

Notice that you were thinking like an Egyptian because you never considered 1/6 or divided 5 by 1/6! You simply had 5 copies of 6 or 30 loaves

Multiplication of Whole Numbers

But in fact, the Egyptian understanding of this problem is *not* multiplication with $5 \times 6 = 30$.

A scribe does not have the concept of multiplying a number by 5. They can only recursively multiply by 2 by considering the sum of a number and itself. That is, we define $2 \times x$ to be $x + x$ for any number x.

Example: "Multiplication by 2" would correspond to summing a number with itself twice. So "multiply 2 by 3" would be found by taking 3 and adding it to itself to get 6:

$$(2 \times 3) \rightarrow 3 + 3 = 6$$

Multiplication of Whole Numbers

Example: "Multiply 4 by 5" is the same as "multiply 2 by 2 by 5" and would be found by taking 5, adding it to itself to get 10, writing down 10, and then adding 10 to itself to get 20:

$$4 \times 5 = 2 \times (2 \times 5) \rightarrow 2 \times (5 + 5) = 2 \times 10 \rightarrow 10 + 10 = 20$$

Share with Friends:

How would you "multiply 8 by 2" as an Egyptian?

Is there a difference in the process involved between "multiply 8 by 2" and "multiply 2 by 8"? Is the result the same?

Taking advantage of commutativity, which would you prefer to calculate? Why?

Multiplication of Whole Numbers

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Problem: How would we "multiply 5 by 4" since 5 is not a multiple of 2 and we only know how to multiply 2 by a number?

>Recognize that "multiply 5 by 4" will give the same result as "multiply 4 by 5"

But what about "multiply 5 by 7"? We cannot just switch the order because both 5 and 7 are not multiples of 2. What are your ideas?

Multiplication of Whole Numbers

Big Idea for Today: we will get a combination of powers of 2 to be equal to the number on the left so we know how to multiply

Notice that we can write 5 as $1 + 2^2$ so we actually know how to "multiply 5 by 4" directly by decomposing 5 into a sum of powers of 2 (this is coming from the binary representation of 5 as 101 if you are seeing this!)

$$5 \times 4 = (1 + 2^2) \times 4 \rightarrow 1 \times 4 + 2^2 \times 4$$

$1 \times 4 = 4$ and $2^2 \times 4 = 2 \times (2 \times 4) \rightarrow 2 \times (4 + 4) = 2 \times 8 \rightarrow 8 + 8 = 16$ so

$$5 \times 4 = 1 \times 4 + 2^2 \times 4 = 4 + 16 = 20$$

Multiplication of Whole Numbers

That was complicated but the Egyptians have it streamlined by using the following table:

5		
1	4	✓Use
2	8	
4	16	✓Use
	20	

Multiplication of Whole Numbers

"Multiply 11 by 4"

11		
1	4	✓Use
2	8	✓Use
4	16	
8	32	✓Use
	44	